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Complexity Theory
Winter 2016

Assignment 12

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Exercise 1.

Suppose you are given an alternating Turing machine M. A pair (K_i^x, K_j^x) of configurations of M is called *alternation* if and only if K_i^x is existential and K_j^x is universal or K_i^x is universal and K_j^x is existential.

An alternating Turing machine M is said to be a Σ_k*-machine* if and only if the initial configuration is an existential one and each computation path of $\beta_M(x)$ contains at most k alternations.

Prove that for any language L and any $k \in \mathbb{N}_+$, the following holds:

$$L \in \Sigma_k^p \iff \text{there is a } \Sigma_{k-1}\text{-machine } M \text{ accepting } L \text{ in polynomial time}$$

Exercise 2.

Design a probabilistic polynomial-time Turing machine M satisfying

$$\text{prob}_M(a_1 a_2 \dots a_n) = \sum_{i=1}^{n} a_i \cdot 2^{-i}$$

for $n \in \mathbb{N}_+$, $a_1, a_2, \dots, a_n \in \{0, 1\}$.

Exercise 3.

For $0 \leq \alpha < 1$, define PP_α to be the class of all sets L such that there exists a probabilistic polynomial-time Turing machine M such that

$$x \in L \iff \text{prob}_M(x) > \alpha$$

for all $x \in \Sigma^*$.

(a) Which known class coincides with PP_α?

(b) Define for the probabilistic polynomial-time Turing machine M designed in Problem 2 and $0 \leq \alpha < 1$ the set $A_\alpha =_{\text{def}} \{ x \mid \text{prob}_M(x) > \alpha \}$ which clearly belongs to PP_α. Prove that if $\alpha < \beta$ then $A_\beta \subset A_\alpha$.

(c) Prove, using (b), that there is an α such that PP_α cointains non-decidable sets.