

## Assignment 9

**Issue date:** 21 Dec 2016    **Due date:** 11 Jan 2017

### Exercise 1.

Show that the following statements are true for sets  $A, B \subseteq \{0, 1\}^*$  such that  $A, B \in \text{NP}$ :

- (a)  $A \cap B \in \text{NP}$  (i.e., NP is closed under intersection)
- (b)  $A \cup B \in \text{NP}$  (i.e., NP is closed under union)
- (c)  $A \cdot B \in \text{NP}$  (i.e., NP is closed under concatenation)
- (d)  $A^* \in \text{NP}$  (i.e., NP is closed under iteration)

### Exercise 2.

Find lowest possible complexity classes in the polynomial hierarchy for following problems:

- (a)  $\{ H \mid H \text{ is a propositional formula such that there is no shorter logically equivalent propositional formula } H' \}$
- (b)  $\{ H \mid H \text{ is a satisfiable propositional formula such that the lexicographically smallest satisfying assignment has an even number of 1's } \}$
- (c)  $\{ G \mid G \text{ is an undirected graph having an even chromatic number } \}$

### Exercise 3.

Which complexity classes coincide with  $\text{P}^{\text{E}}$ ,  $\text{LIN}^{\text{P}}$ , and  $\text{P}^{\text{NP} \cap \text{coNP}}$ ?

**Merry Christmas and a Happy New Year!**