University of Konstanz
Department of Computer & Information Science
Prof. Dr. Sven Kosub / Michael Aichem

Complexity Theory
Winter 2016

Assignment 10

Issue date: 11 Jan 2017 **Due date:** 18 Jan 2017

Let $\mathcal{K}$ and $\mathcal{K}'$ be any classes of sets. We consider the following new classes (representing a kind of polynomial logic over complexity classes):

$$\begin{aligned}
\exists\mathcal{K} &=_{\text{def}} \{\ A \mid x \in A \leftrightarrow (\exists^p z)[(x,z) \in B] \text{ for all } x, \text{ appropriate } B \in \mathcal{K}, \text{ polynomial } p\ \} \\
\forall\mathcal{K} &=_{\text{def}} \{\ A \mid x \in A \leftrightarrow (\forall^p z)[(x,z) \in B] \text{ for all } x, \text{ appropriate } B \in \mathcal{K}, \text{ polynomial } p\ \} \\
\text{co}\mathcal{K} &=_{\text{def}} \{\ \overline{A} \mid A \in \mathcal{K}\ \} \\
\mathcal{K} \wedge \mathcal{K}' &=_{\text{def}} \{\ A \cap B \mid A \in \mathcal{K},\ B \in \mathcal{K}'\ \} \\
\mathcal{K} \vee \mathcal{K}' &=_{\text{def}} \{\ A \cup B \mid A \in \mathcal{K},\ B \in \mathcal{K}'\ \}
\end{aligned}$$

Exercise 1.

Prove the following equalities:

(a) $\exists\text{NP} = \text{NP}$

(b) $\forall\text{coNP} = \text{coNP}$

Exercise 2.

Which pairwise inclusion-relationships hold for the following five classes?

$$\text{NP} \cup \text{coNP}, \quad \text{NP} \vee \text{coNP}, \quad \text{NP} \wedge (\text{NP} \cap \text{coNP}), \quad \text{NP}, \quad \text{NP} \wedge (\text{NP} \cup \text{coNP})$$

Exercise 3.

Which complexity classes in the polynomial hierarchy are captured by the following classes?

(a) $\exists(\text{NP} \wedge \text{coNP})$

(b) $\forall(\text{NP} \wedge \text{coNP})$

(c) $\exists(\text{NP} \vee \text{coNP})$

(d) $\forall(\text{NP} \vee \text{coNP})$

(e) $\forall(\text{NP} \vee \exists(\text{coNP} \wedge \forall\text{NP}))$