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Complexity Theory
Winter 2016

Assignment 4

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Exercise 1.

For a function $r : \mathbb{N} \to \mathbb{N}$ satisfying $r(n) > n$ for all $n \in \mathbb{N}$, a language $A \subseteq \Sigma^*$, and distinct letters $a, b \in \Sigma$, we define the *padding language* A_r as follows:

$$A_r =_{\text{def}} \{\ xba^{r(|x|)-|x|-1} \mid b \neq a \wedge x \in A\ \}$$

A function $t : \mathbb{N} \to \mathbb{N}$ is said to be *time-constructible* if and only if the mapping $x \mapsto t(|x|)$ is computable in time $t(|x|)^k$ for some $k \geq 1$ on a T-TM.

Show that for any time-constructible functions $t(n) \geq n$ and $r(n) > n$, the following is true:

$$A \in \text{DTIME}(\text{Pol}\ t \circ r) \iff A_r \in \text{DTIME}(\text{Pol}\ t)$$

Exercise 2.

Show that the following statements hold for all functions f, g:

(a) If $f, g \in \text{FL}$ then $f \circ g \in \text{FL}$.

(b) If $f, g \in \text{FP}$ then $f \circ g \in \text{FP}$.

Exercise 3.

For a word function $f : \Sigma^* \to \Delta^*$, we define

$$\text{Symb}(f) =_{\text{def}} \{\ (x, m, f(x)_m) \mid x \in \Sigma^*, m \geq 1\ \},$$

where z_m denotes the m-th symbol in $z \in \Delta^*$ (z_m is not defined for $m \notin \{1, \ldots, |z|\}$).

Show that for each monotone function $s(n) \geq \log n$ the following holds:

$$f \in \text{FDSPACE}(\text{Pol}\ s) \iff \text{Symb}(f) \in \text{DSPACE}(\text{Pol}\ s) \text{ and } |f(x)| \leq_{\text{ae}} 2^{s(|x|)^k} \text{ for some } k$$