University of Konstanz
Department of Computer & Information Science
Prof. Dr. Sven Kosub / Michael Aichem

Complexity Theory
Winter 2016

Assignment 6

Issue date: 30 Nov 2016 **Due date:** 07 Dec 2016

Exercise 1.

Let CFL denote the class of all context-free languages. Show that

$$\mathrm{CFL} \subseteq \mathrm{DSPACE}\left((\log n)^2\right).$$

Apply the following lemma (without proving it):

> **Lemma.** Let $G = (\Sigma, N, S, R)$ be a context-free grammar. For each derivation $A \Rightarrow_G^* w$, there exist a nonterminal $B \in N$ and words w_1, w_2, w_3 satisfying $w = w_1 w_2 w_3$ and $\frac{1}{3}|w| \leq |w_2| \leq \frac{2}{3}|w|$ such that $A \Rightarrow_G^* w_1 B w_3$ and $B \Rightarrow_G^* w_2$.

Hint: Use the divide-and-conquer technique as in the proof of Savitch's theorem.

Exercise 2.

Show that there exist $\leq_m^{\log}$-complete languages for NEXP.

Modify the proof of the existence of $\leq_m^{\log}$-complete languages for NP appropriately.

Exercise 3.

Show that the following is true for arbitrary sets $A \subseteq \Sigma^*$:

$A \in \mathrm{NP} \iff$ there exist a set $B \in P$ and a (monotone) polynomial p such that

$$x \in A \leftrightarrow (\exists y)[|y| = p(|x|) \wedge (x, y) \in B]$$

for all $x \in \Sigma^*$