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Assignment 1

Issue date: 11 May 2017 **Due date:** 18 May 2017

Exercise 1.

Prove or disprove the following logical equivalences:

(a) $\psi \wedge (\varphi \rightarrow \eta) \equiv (\psi \wedge \varphi) \rightarrow \eta$

(b) $\psi \rightarrow (\varphi \wedge \eta) \equiv (\psi \rightarrow \varphi) \wedge (\psi \rightarrow \eta)$

(c) $(\psi \wedge \varphi) \rightarrow \eta \equiv (\psi \rightarrow \eta) \wedge (\varphi \rightarrow \eta)$

(d) $(\psi \vee \varphi) \rightarrow \eta \equiv (\psi \rightarrow \eta) \wedge (\varphi \rightarrow \eta)$

Exercise 2.

Prove the following interpolation theorem of propositional logic:

> Let $(\varphi \rightarrow \psi) \in \mathsf{PL}$ be a tautology. Then, there exists a formula $\eta \in \mathsf{PL}$ such that $\tau(\eta) \subseteq \tau(\varphi) \cap \tau(\psi)$ and both $(\varphi \rightarrow \eta)$ and $(\eta \rightarrow \psi)$ are tautologies.

Hint: Use induction on the number of variables occuring in φ but not in ψ.

Exercise 3.

Find appropriate sequences of derivations showing the following statements:

(a) $\{\varphi\} \vdash \varphi \wedge (\psi \vee \varphi)$

(b) $\{\neg\neg\varphi\} \vdash \varphi$

(c) $\emptyset \vdash (\varphi \rightarrow \psi) \rightarrow (\neg\psi \rightarrow \neg\varphi)$

(d) $\{\varphi \wedge \psi\} \vdash \neg(\neg\varphi \vee \neg\psi)$