

Assignment 3

Issue date: 29 Jun 2017 Due date: 06 Jul 2017

Exercise 7.

Find classes (possibly low) in the arithmetical hierarchy for each of the following problems:

- (a) $\text{TOT} =_{\text{def}} \{ i \mid \text{RAM } M_i \text{ always stops} \}$
- (b) $\text{NU} =_{\text{def}} \{ i \mid \text{RAM } M_i \text{ never stops} \}$
- (c) $\text{FIN} =_{\text{def}} \{ i \mid \text{RAM } M_i \text{ stops on finitely many inputs} \}$
- (d) $\text{COFIN} =_{\text{def}} \{ i \mid \text{RAM } M_i \text{ does not stop on finitely many inputs} \}$

Exercise 8.

Let τ be a vocabulary. A *theory* T is a set of $\text{FO}(\tau)$ formulas such that $T = T^{\models}$ and T is satisfiable. A theory T is said to be *axiomatizable* if and only if there exists a decidable set $\Phi \subseteq \text{FO}(\tau)$ such that $\Phi^{\models} = T$.

Prove the following statements for a theory T :

- (a) If T is decidable then T is axiomatizable.
- (b) If T is axiomatizable then T is enumerable.
- (c) If T is complete and enumerable then T is decidable.

Exercise 9.

Complete the proof of Church's theorem as discussed in the lecture. In particular, find subformulas for the following two types of RAM instructions:

- (a) **GOTO** k
- (b) **IF** $Rk=0$ **GOTO** ℓ