

## Assignment 2

Issue date: 02 Nov 2016 Due date: 09 Nov 2016

### Exercise 1.

Define  $S =_{\text{def}} \{ ww^R \mid w \in \{0,1\}^* \}$ . Show that the following holds for all  $\varepsilon > 0$ :

- (a)  $\overline{S} \in 1\text{-T-NTIME}(n)$
- (b)  $\overline{S} \in 1\text{-T-NSPACE}(\varepsilon \cdot \log n)$

### Exercise 2.

A function  $s : \mathbb{N} \rightarrow \mathbb{N}$  is said to be *space-constructible* if and only if there is a 2-T-TM  $M$  such that  $\text{DSPACE}_M(x) = s(|x|)$  for all  $x \in \Sigma^*$ .

Show that the following functions are space-constructible:

- (a)  $s : n \mapsto n^k$  for  $k \geq 1$
- (b)  $s : n \mapsto \log n$
- (c)  $s : n \mapsto 2^n$

### Exercise 3.

Let  $s, s' : \mathbb{N} \rightarrow \mathbb{N}$  be space-constructible (for a definition see Exercise 2). Show that the following functions are space-constructible:

- (a)  $\max(s, s') : n \mapsto \max(s(n), s'(n))$
- (b)  $s + s' : n \mapsto s(n) + s'(n)$
- (c)  $s \cdot s' : n \mapsto s(n) \cdot s'(n)$
- (d)  $s \circ s' : n \mapsto s(s'(n))$  for  $s(n) \geq n$