University of Konstanz
Department of Computer & Information Science
Prof. Dr. Sven Kosub / Michael Aichem

Complexity Theory
Winter 2016

Assignment 7

Issue date: 07 Dec 2016 **Due date:** 14 Dec 2016

Exercise 1.

Prove that $(3,4)$-SAT is $\leq_m^{\log}$-complete for NP.

Exercise 2.

The set of *integer expressions* is inductively defined as follows: For each $n \in \mathbb{N}$ is $\mathrm{dya}(n)$ an integer expression; if H' and H' are integer expression then $(H + H')$ and $(H \cup H')$ are integer expressions.

The finite set $L(H) \subseteq \mathbb{N}$ given by an integer expression H is accordingly defined as follows:

$$\begin{aligned} L(\mathrm{dya}(n)) &=_{\mathrm{def}} \{n\} \\ L(H + H') &=_{\mathrm{def}} \{\ n + m \mid n \in L(H), m \in L(H')\ \} \\ L(H \cup H') &=_{\mathrm{def}} L(H) \cup L(H') \end{aligned}$$

Determine the sets $L(H)$ for the following integer expressions:

(a) $((12 \cup 21) + 111)$

(b) $((12 + 21) \cup 111)$

(c) $(((1 \cup 2) + 11) \cup 12)$

(d) $(((((2 + 2) + 2) + 2) + 2)+2)$

Exercise 3.

Prove that the set

$$\text{IE-Member} =_{\mathrm{def}} \{\ (H, n) \mid H \text{ is an integer expression, } n \in \mathbb{N} \text{ and } n \in L(H)\ \}$$

is $\leq_m^{\log}$-complete for NP.

Hint: Show that Subset Sum $\leq_m^{\log}$ IE-Member.