University of Konstanz
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Logic in Computer Science
Summer 2017

Assignment 2

Issue date: 06 Jun 2017 **Due date:** 13 Jun 2017

Exercise 4.

Let τ be some vocabulary containing the unary relation symbols P, Q.

Prove or disprove the following logical equivalences:

(a) $\exists x(\varphi \vee \psi) \equiv \exists x\varphi \vee \exists x\psi$

(b) $\forall x(\varphi \vee \psi) \equiv \forall x\varphi \vee \forall x\psi$

(c) $\varphi \vee \forall x\psi \equiv \forall x(\varphi \vee \psi)$ if x does not occur in φ

(d) $\forall x(Px \vee Qx) \equiv Px \vee \forall xQx$

Exercise 5.

Let τ be any vocabulary. An $\mathsf{FO}(\tau)$ formula φ is said to be in *prenex normal form* if and only if $\varphi = Q_1x_1 \ldots Q_rx_r\psi$ where $Q_i \in \{\exists, \forall\}$ and ψ is quantifier-free.

Find for each of the following $\mathsf{FO}(\tau)$ formulas a logically equivalent $\mathsf{FO}(\tau)$ formula in prenex normal form (for $P, Q, R \in \tau$):

(a) $\forall x\exists yPxy \vee (\neg Qz \wedge \neg\exists xRxy)$

(b) $\exists yRxy \leftrightarrow \forall xRxx$

Exercise 6.

Let τ be any vocabulary.

Prove the following statements for sets $\Phi \subseteq \mathsf{FO}(\tau)$ and $\mathsf{FO}(\tau)$ formulas φ:

(a) If $\varphi \notin \Phi^{\vdash}$ then $\Phi \cup \{\neg\varphi\}$ is consistent.

(b) If $\varphi \in \Phi^{\vdash}$ and Φ is consistent then $\Phi \cup \{\varphi\}$ is consistent.

(c) Φ is consistent if and only if $\Phi \cup \{\varphi\}$ is consistent or $\Phi \cup \{\neg\varphi\}$ is consistent.