

Assignment 5

Issue date: 23 Nov 2016 **Due date:** 30 Nov 2016

Exercise 1.

Exercise shifted to Assignment 6!

Let CFL denote the class of all context-free languages. Show that

$$\text{CFL} \subseteq \text{DSPACE}((\log n)^2).$$

Apply the following lemma (without proving it):

Lemma. Let $G = (\Sigma, N, S, R)$ be a context-free grammar. For each derivation $A \Rightarrow_G^* w$, there exist a nonterminal $B \in N$ and words w_1, w_2, w_3 satisfying $w = w_1 w_2 w_3$ and $\frac{1}{3}|w| \leq |w_2| \leq \frac{2}{3}|w|$ such that $A \Rightarrow_G^* w_1 B w_3$ and $B \Rightarrow_G^* w_2$.

Hint: Use the divide-and-conquer technique as in the proof of SAVITCH's theorem.

Exercise 2.

Show that for each function $s : \mathbb{N} \rightarrow \mathbb{N}$ the following statements are equivalent:

- (1) s is space-constructible.
- (2) $x \mapsto s(|x|) \in \text{FSPACE}(s)$.
- (3) $x \mapsto 2^{s(|x|)} \in \text{FSPACE}(s)$.

Exercise 3.

Show that the following implication is true:

$$\text{LIN} \subseteq \text{NP} \implies \text{NP} = \text{PSPACE}.$$

Hint: Use the padding technique.